The Simple Economics of Affirmative Action Policies: Some Theory and Some Evidence

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Discussion Group on Affirmative Action

Week 2: September 21, 2018

Institute for Advanced Study in Toulouse

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(2) Optimal design of AA programs (Fryer and Loury, JPE 2013)

(3) Simulated impact of "color-blindness" using US college data (Fryer, Loury, Yuret, JLE0, 2007)

(1) The Problem of Persistent Group Inequality:

One can show that historical group disparities may persist even with equal market opportunities when intergenerational HC spillovers and social segregation are sufficiently important.

Race: America’s Seemingly Permanent Dilemma

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- A large scale non-European immigration since 1964 has shifted the social/political landscape on racial inequality issues.

- While progress has been made raising blacks’ low social/economic status, we have NOT solved this problem, and are now in danger of losing our way, I fear.

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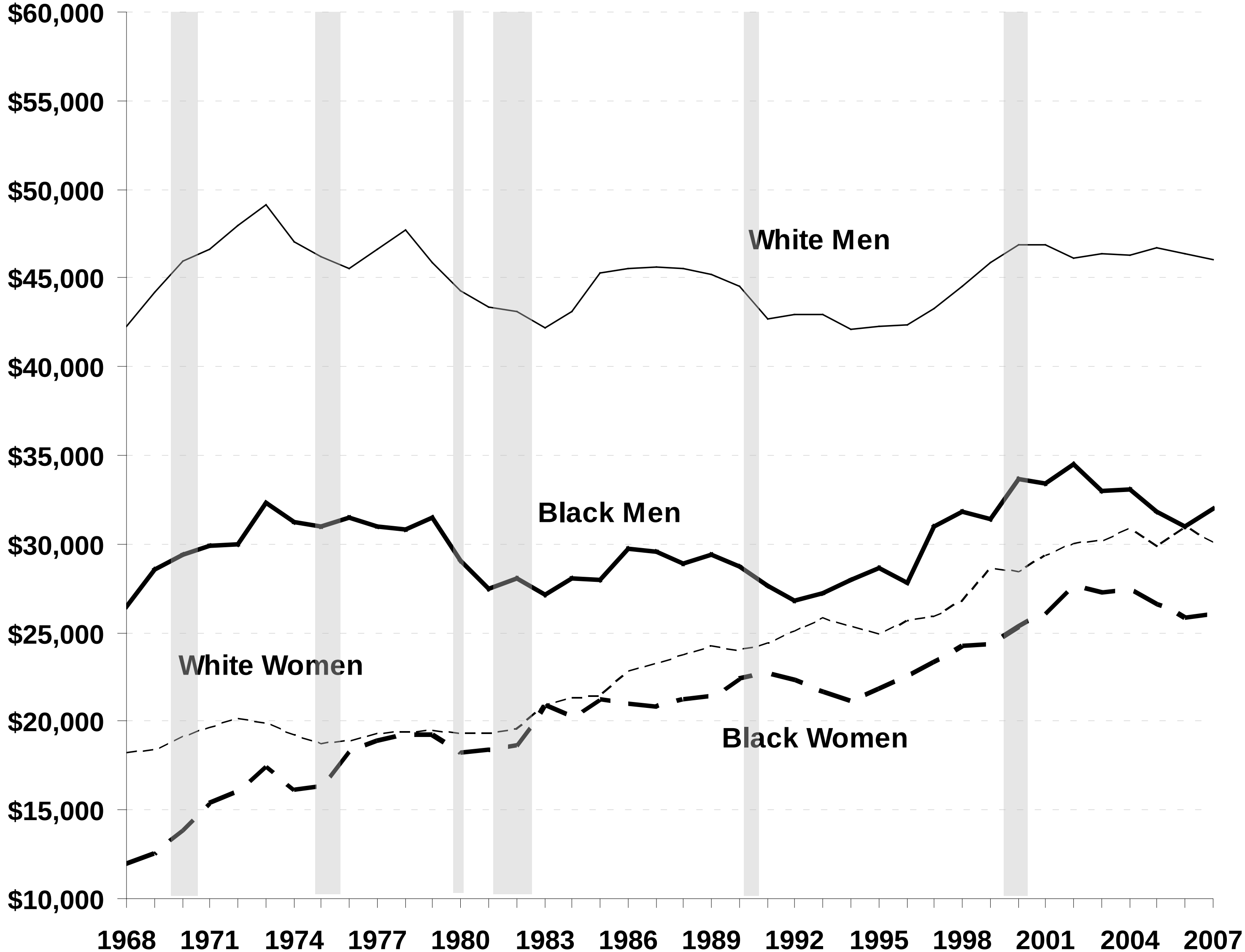
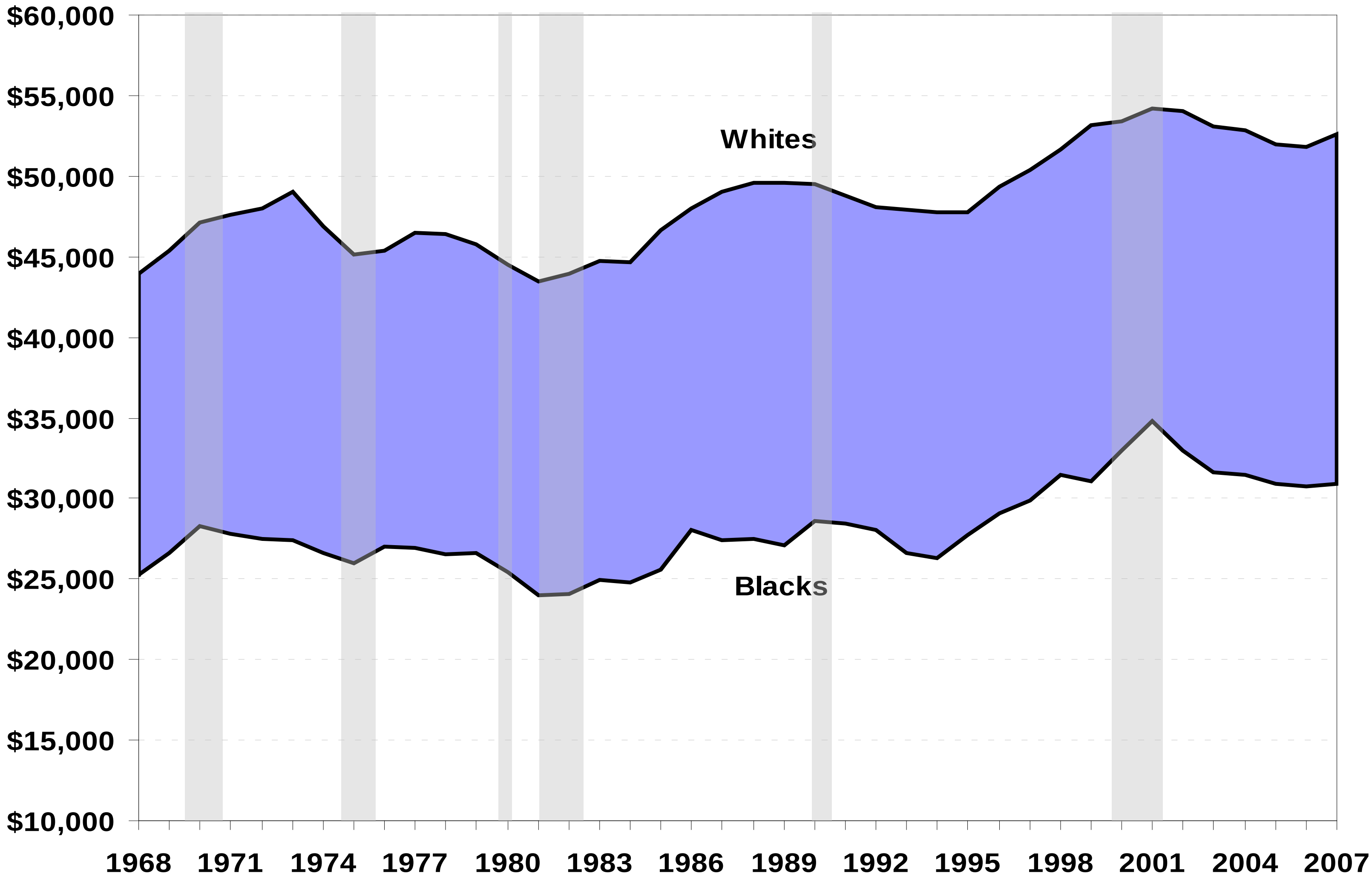
- Consider some social/economic trends in the period1968 - 2008

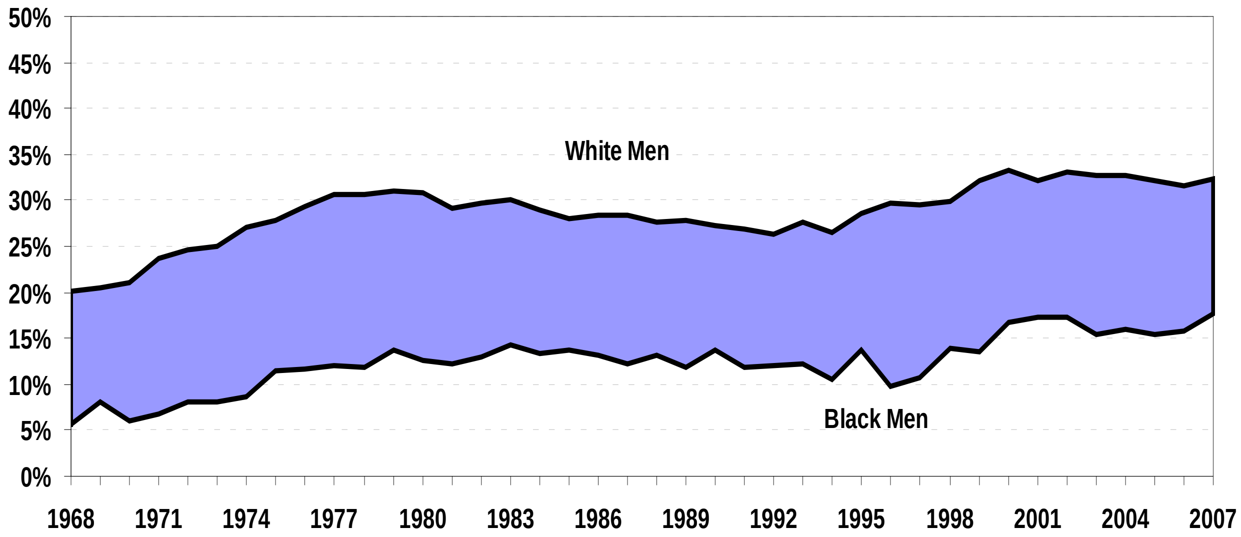
Median Wage and Salary Earnings for Native-Born Non-Hispanics Reporting Earnings

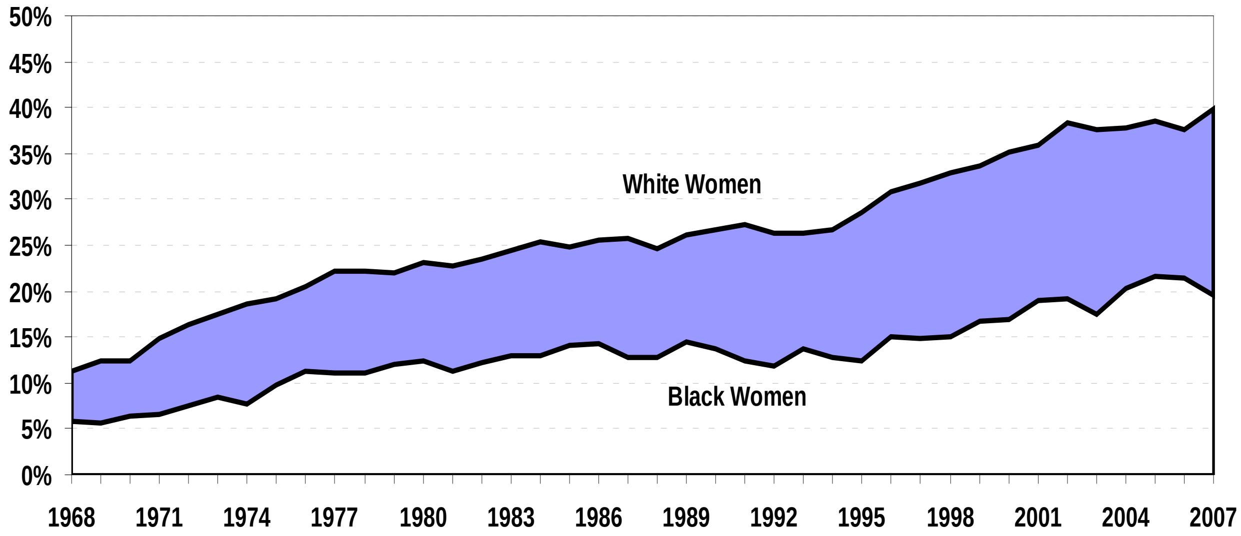


Median Income of Households Headed by Native-Born Non-Hispanics
(shown in constant 2007 Dollars)

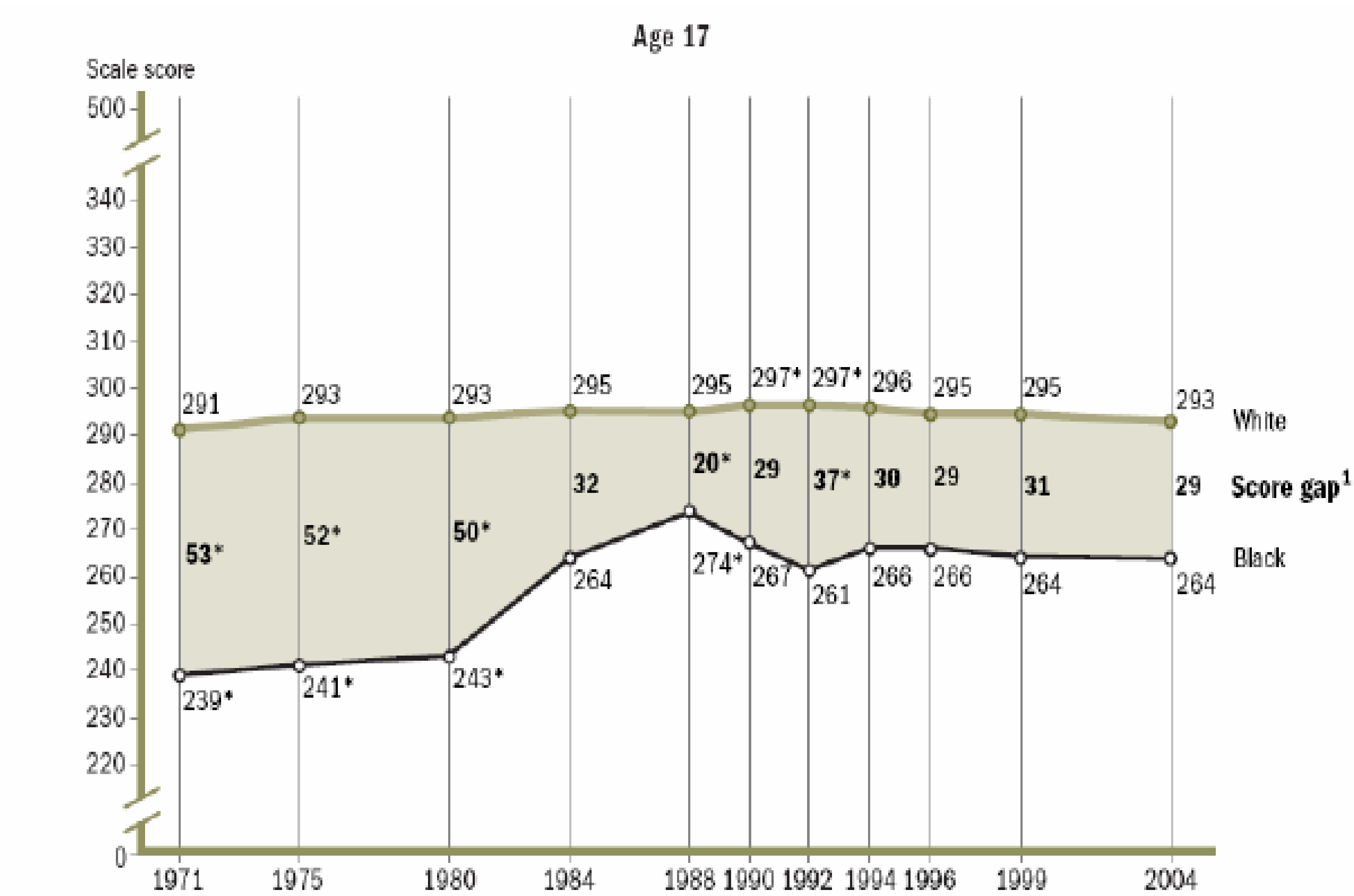


Percent of Native-Born, Non-Hispanic Men and Women Aged 25 to 34 Reporting a Four-Year College Education

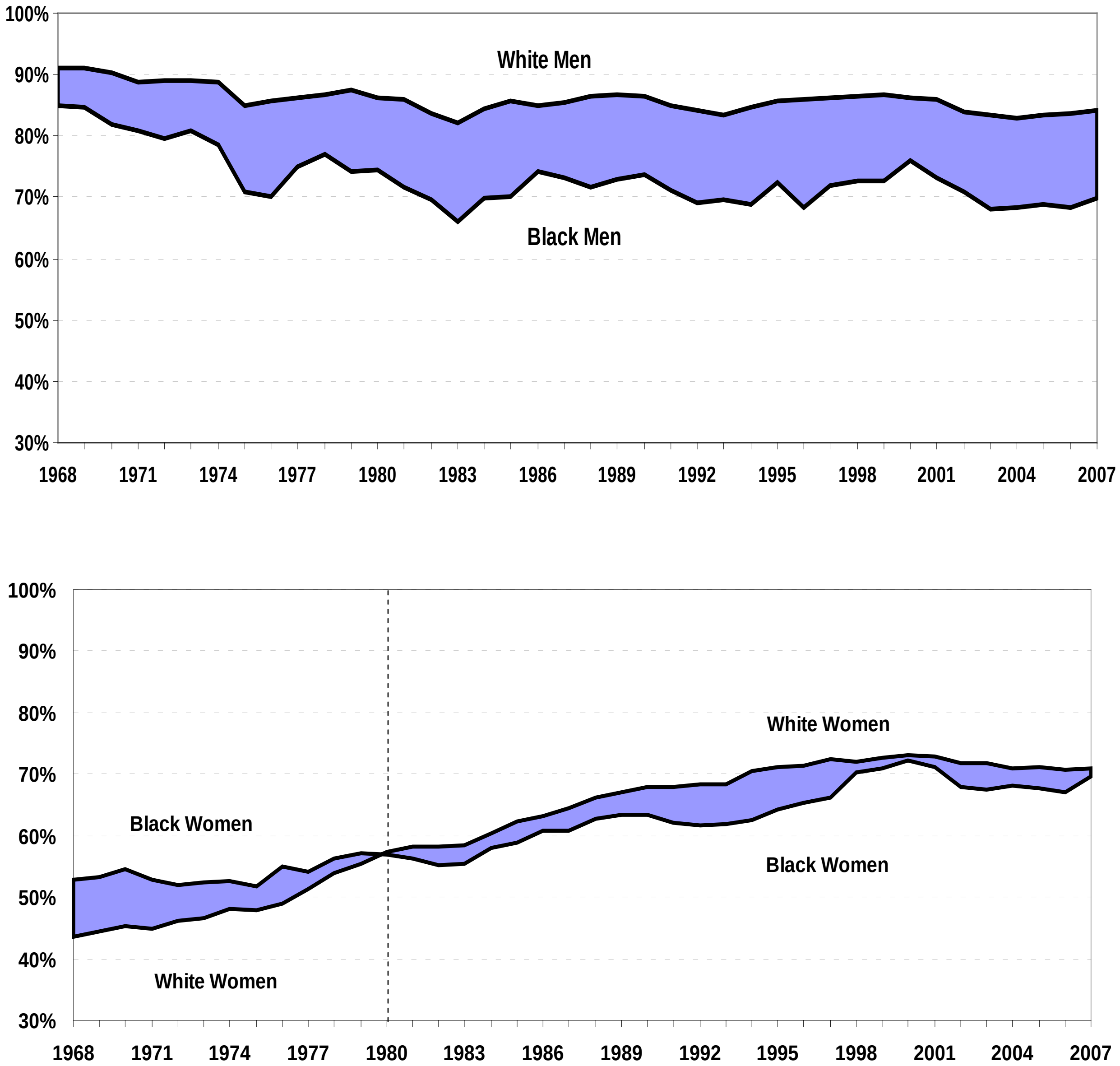




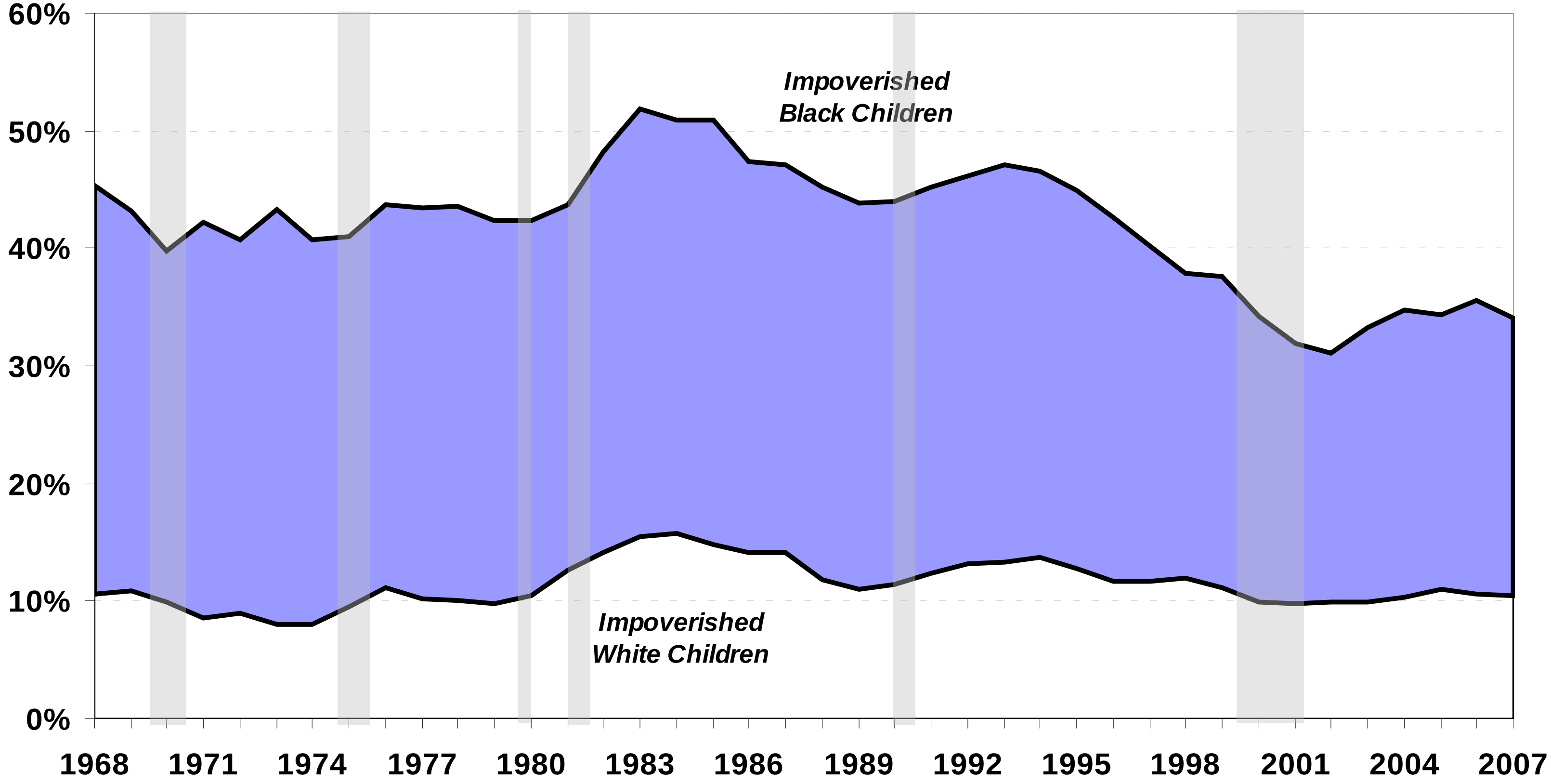
Reading Scores (National Assessment of Educational Progress)



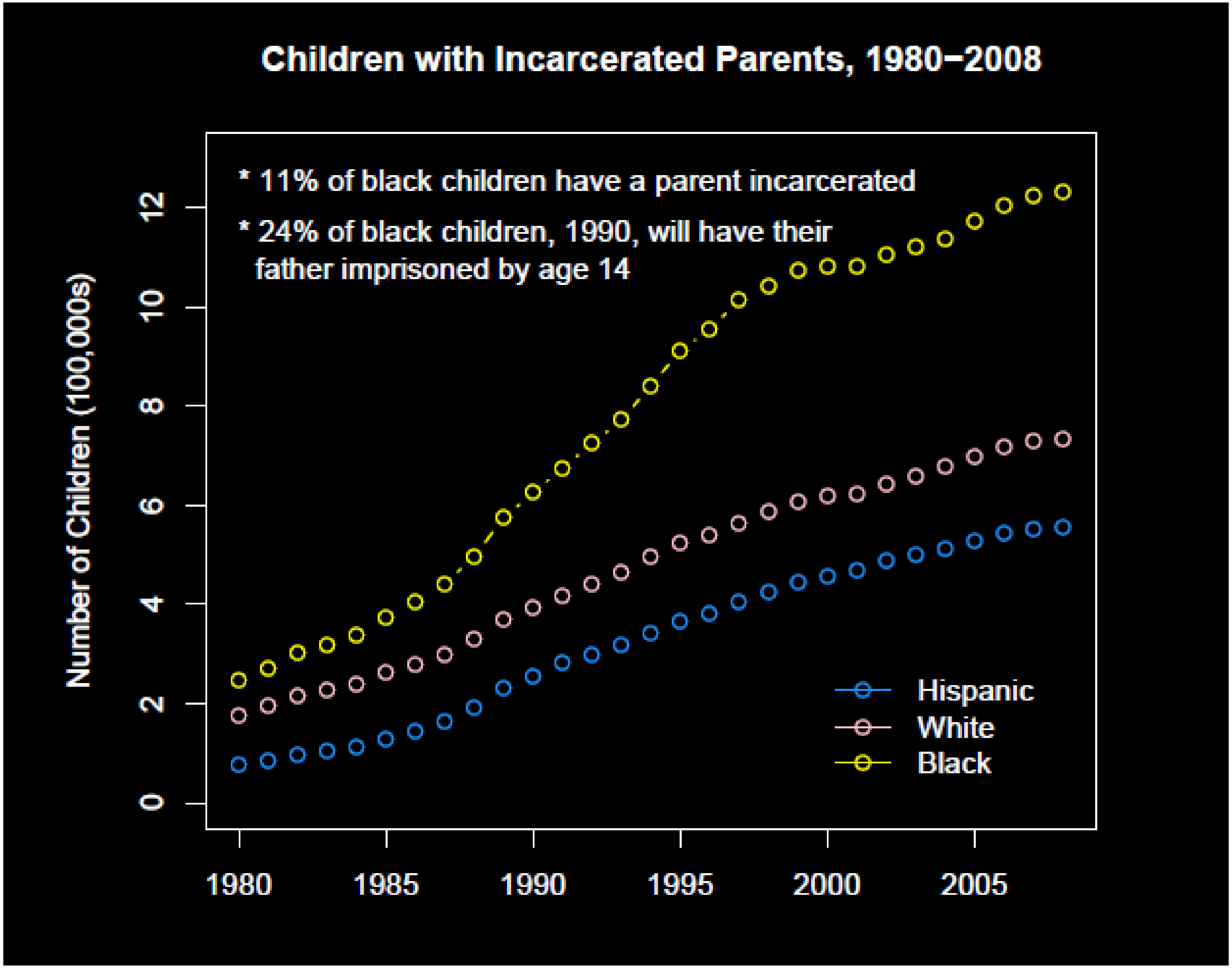
Percent of Native-Born, Non-Hispanic Men and Women Aged 25 to 59 Employed; 1968 to 2007



Percent of Native-Born Non-Hispanic Children Under Age 18
Living Below the Poverty Line; 1968 to 2007



The Prison Intersects with Families and Communities. Note Incarceration's Huge Impact of Black Children.



Illustrating How Social Segregation and Behavioral Spillovers Can Lead to Persistent Group Inequality

Eastern Economic Journal, **2013**, **39**, (425–438)

www.palgrave-journals.com/eej/

Presidential Address

The Superficial Morality of Color Blindness: Why "Equal Opportunity" May Not Be Enough?

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aptara	JEEA	jeea12037	Dispatch: August 5, 2013	CE: N/A
	Journal	MSP No.	No. of pages: 24	PE: Grily Chua

GROUP INEQUALITY

Samuel Bowles
Santa Fe Institute and the University of Siena

Rajiv Sethi
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• Each young person has ties to a large number of older people, and the fraction of "out-group" ties depends on degree of segregation.

• A demographic parameter $\beta \in (0,1)$ denotes the relative number of group B agents in each generation. (So if $\beta < 1/2$ then disadvantaged are a minority of the overall population, etc.)

- A segregation parameter $\eta \in (0,1)$ denotes the probability that a young person's social tie is to some old person drawn at random from within his same social group. And $1-\eta$ is the chance a tie is drawn at random from overall old population. ($\eta=1$ implies total segregation.)

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• ***Key Behavioral Assumption***: There exist a quality threshold $\sigma^* \in (0,1)$ such that a young person adopts the "decent" orientation if and only if the quality of his social influences, σ, exceeds this threshold.

• How does the "decent vs. street" conflict evolve over time in this model, given demographic/segregation parameters σ and η? Note that "everybody decent" and "everybody street" are both stable behavioral configurations in this society. More interesting is that "all A's decent, all B's street" is also stable behavioral configuration if η is big enough!

SOCIAL STRUCTURE

* We assume that the quality of an agent's social network depends on group identity and generation of birth.

* Specifically, an agent born at date $t+1$ has a large number of socia ties to generation t agents.

* Each of these ties is, with probability $\eta \in [0,1]$, drawn at random from the agent's social group (A or B).

* With probability $1-\eta$ the associate is drawn at random from among the general population of agents without regard to group identity.

* Let x_i^t be the fraction of generation t agents in group i who become high skilled, and let σ_i^{t+1} denote the quality of the social network of a generation $t+1$ agent in group i. Then:

$$\sigma_i^{t+1} = \eta x_i^t + (1-\eta)[(1-\beta)x_a^t + \beta x_b^t]$$

* So, the probability that an associate of a group A agent belongs to group A equals

$$\eta + (1 - \eta)(1 - \beta) \equiv \alpha_1$$

* While, the probability that an associate of a group B agent belongs to group A equals

$$(1 - \eta)(1 - \beta) \equiv \alpha_0$$

Thus, once adopted, a behavioral configuration in this society where all A's adopt a "decent" orientation and all B's adopt a "street" orientation would tend to persist across the generations whenever $\alpha_1 > \sigma^* > \alpha_0$

Theorem 1: There exists a minimal degree of in-group bias in associational behavior, $\underline{\eta}(\beta,\sigma^*)$, such that whenever $\eta > \underline{\eta}(\beta,\sigma^*)$ ("social segregation") then the initial condition of gorup inequality $(x_a^0, x_b^0) = (1,0)$ is a stable steady state equilibrium. Moreover,

$$\underline{\eta}(\beta,\sigma^*) \equiv Max\{1 - \frac{\sigma^*}{1-\beta}; 1 - \frac{1-\sigma^*}{\beta}\}$$

Furthermore, when $\eta < \underline{\eta}(\beta,\sigma^*)$ ("social integration") then the system con-

verges, from the initial state $(x_a^0, x_b^0) = (1,0)$, in one period to a steady state with group equality. This steady state is "skill-enhancing" (relative to the initial condition) when the disadvantaged group is not too big ($\beta < 1-\sigma^*$) and it is "skill-reducing" when $\beta < 1-\sigma^*$.

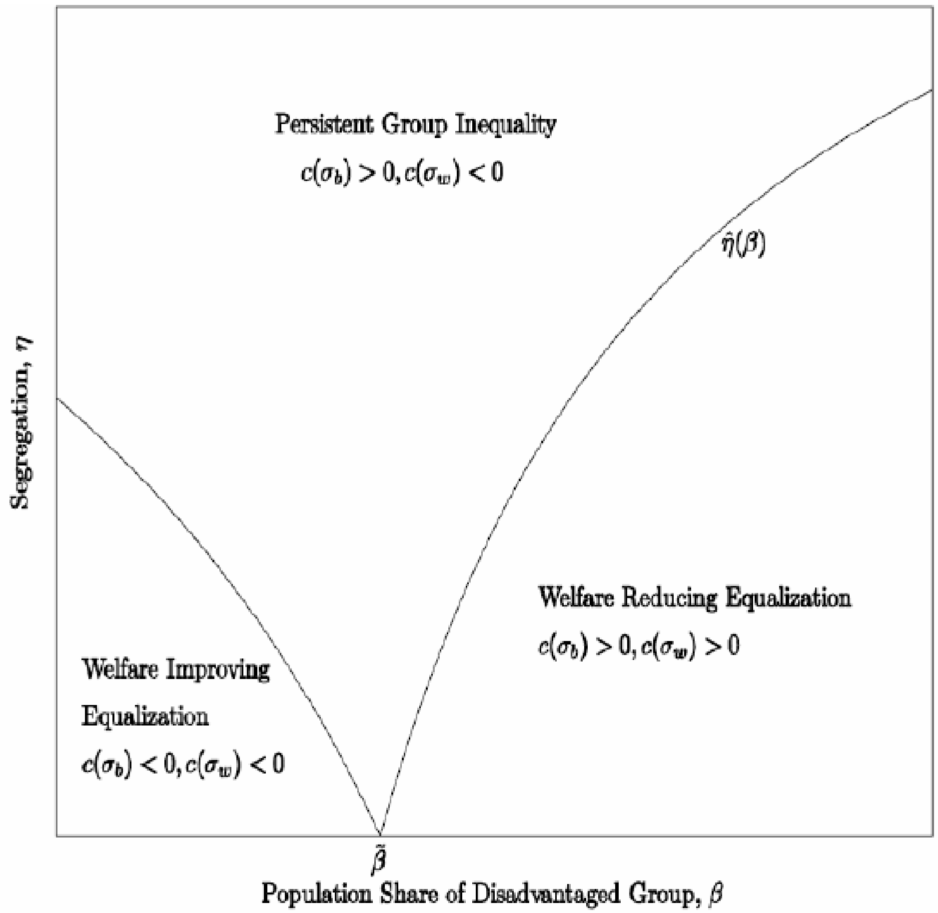

Figure 2. Effects of segregation and population shares on persistent inequality

(2) A Theory of Optimal Affirmative Action Policies:

“Blind” vs. “Sighted” Policy operating at the Development Margin vs. the Assignment Margin

Valuing Diversity

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Harvard University

Glenn C. Loury

Brown University

[*Journal of Political Economy*, 2013, vol. 121, no. 4]

Dimensions of AA Policy Design

We distinguish affirmative action policies along two abstract dimensions: Sightedness and Timing

Sightedness refers to whether or not the policy is group-blind – that is, whether or not information on an individual's group identity is required in order to implement the policy.

Timing refers to the stage in the economic process at which an AA policy operates – the *development margin*, where skills are being acquired; and/or the *assignment* margin, where opportunities for productive activity are being allocated.

Questions of Interest about AA Policy

- How does affirmative action policy affect incentives to acquire skils?
- Is affirmative action best undertaken early or late in the skill development process?
- What impact does a “blindness” constraint have on the design and efficiency of AA?

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- A continuum of productive ‘slots,’ but fewer slots than workers.
- At a preliminary stage (development margin) workers decide to acquire costly HC or not.
- HC costs vary among workers and are distributed differently as between groups. B’s are “disadvantaged” (have higher costs.)
- At a final stage (assignment margin) scarce slots are allocated among the more numerous workers.

Theoretical Model Description (Continued)

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- AA policy is modeled here as a tax/subsidy intervention at either the development margin, the assignment margin, or both.

- Because cost distributions differ, the presence of groups among slot holders also differ, absent intervention. AA can remedy this.

Notation:

$i \in \{A,B\}$ – two identity groups; $\lambda_i \in (0,1)$ –fraction of workers in group i. $[\lambda_a + \lambda_b = 1]$

$e \in \{0,1\}$ – HC investment choice (development margin). $c \geq 0$ – worker's cost of HC investment.

$G_i(c)$ = CDF of cost distribution in group i. $G(c) = \lambda_a G_b(c) + \lambda_b G_b(c)$ = CDF of cost distribution for the overall population. (Let $g_i(c)$ = the associated PDF, etc.)

$\pi \in [0,1]$ – the fraction of some worker population with HC choice $\{e=1\}$.

$\mu \geq 0$ – a worker's productivity = that worker's output if he obtains a slot.

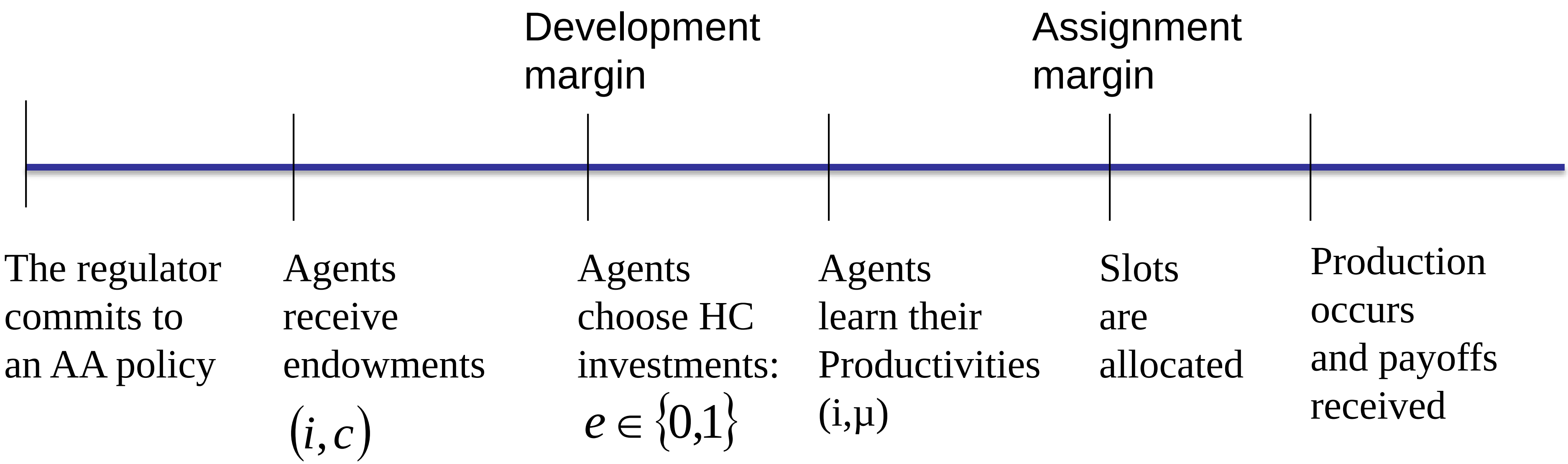
$F_e(\mu)$ = a worker's productivity CDF given that his HC choice is $e \in \{0,1\}$. (Let $f_e(\mu)$ = the associated PDF, etc.)

$F(\pi,\mu) = \pi F_1(\mu) + (1-\pi)F_0(\mu)$ = prod. CDF in population if fraction π choose $\{e=1\}$.

$\theta \in (0,1)$, inelastic supply of slots; $p \geq 0$ the price of a slot; $a \in [0,1]$ = probability some worker is assigned a slot (when slots are randomly rationed).

[Note: workers with productivity μ, HC investment cost c, facing slots price p, and making HC investment choice e, receives a payoff = $\text{Max}\{\mu - p; 0\} - c*e$

(1) Development vs. Assignment Margin
(2) Blind vs. Sighted preferential policy.



Note: A “policy” here means a system of subsidies/taxes on agents at both margins, depending on the agents’ actions and (under “sightedness”) their group.

Figure 1: Sequence of Actions

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1. Absent AA, the CE is “efficient”; the disadvantaged group acquires less HC; so it is “underrepresented” in slot-holding relative to its presence in the population.
2. The optimal “sighted” AA policy entails intervention favoring disadvantaged group at the development margin, but NOT at the assignment margin.
3. The optimal “blind” AA policy randomly rations slots at the assignment margin; and it entails a (common) HC subsidy to both groups at development margin if and only if disadvantaged workers are relatively more numerous there than on the assignment margin.

Assume that group B is exogenously "disadvantaged" relative to group A in the following sense:

ASSUMPTION 1. $g_a(c)/g_b(c)$ is a strictly decreasing function of c.

Monotonicity of this likelihood ratio implies that, for c interior to the cost support, (1) $G_a(c) > G_b(c)$, (2)

$$\frac{G_a(c)}{G_b(c)} > \frac{g_a(c)}{g_b(c)} > \frac{1 - G_a(c)}{1 - G_b(c)},$$

and (3) $G_a(c)/G_b(c)$ and $[1 - G_a(c)]/[1 - G_b(c)]$ are both strictly decreasing functions of c.

To counter this "disadvantaged," AA policy aims to increase the presence of group B members among those holding slots

Assume those acquiring HC (e=1) are statistically more productive in the following sense:

ASSUMPTION 2. $f_1(\mu)/f_0(\mu)$ is a strictly increasing function of μ.

As before, monotonicity implies that, for μ interior to the productivity support, (1) $F_1(\mu) < F_0(\mu)$, (2)

$$\frac{F_1(\mu)}{F_0(\mu)} < \frac{f_1(\mu)}{f_0(\mu)} < \frac{1 - F_1(\mu)}{1 - F_0(\mu)},$$

and (3) $F_1(\mu)/F_0(\mu)$ and $[1 - F_1(\mu)]/[1 - F_0(\mu)]$ are both strictly increasing functions of μ.

So, B's can get more slots if more of them acquire HC (the development margin) or if the effective price they must pay for slots is reduced (the assignment margin)

(A) Equilibrium without and AA policy:

The fraction of a population willing to purchase a slot at price p when the fraction π of that population have acquired HC is:

$$1 - F(\pi, p) = \pi[1 - F_1(p)] + (1 - \pi)[1 - F_0(p)]$$

The supply of slots is assumed fixed exogenously at θ. So, for the slots market to clear we must have:

$$1 - F(\pi, p) = \theta$$

Or, equivalently:

$$p = F^{-1}(\pi, 1-\theta)$$

(That is, the top θ quantiles of the productivity distribution get slots, and slots are priced at the productivity of the marginal slot holder.)

At the same time, the return to acquiring HC when p is the anticipated slot price is just the expected rent, B(p), given by:

$$B(p) \equiv \int_p^{\infty} (\mu - p)[f_1(\mu) - f_0(\mu)]\,d\mu$$

$$= \int_p^{\infty} [F_0(\mu) - F_1(\mu)]\,d\mu$$

$$\equiv \int_p^{\infty} \Delta F(\mu)\,d\mu,$$

Since it is rational to acquire HC if and only if the cost does not exceed the benefit, we must have in equilibrium that:

$\Pi = G(B(p))$ or equivalently $B(p) = G^{-1}(\pi)$

(That is, the lowest π quantiles of the cost distribution invest in HC, with cost of the marginal investor just equal to the return.)

Combining these two requirements (slot price = productivity at the (1-θ)th quantile of the productivity distribution; return on HC = investment cost at the πth quantile of cost distribution) we conclude that, in equilibrium, a fraction π^e acquire HC, where:

$$G^{-1}(\pi^e) = \int_{F^{-1}(\pi^e, 1-\theta)}^{\infty} \Delta F(\mu)\, d\mu$$

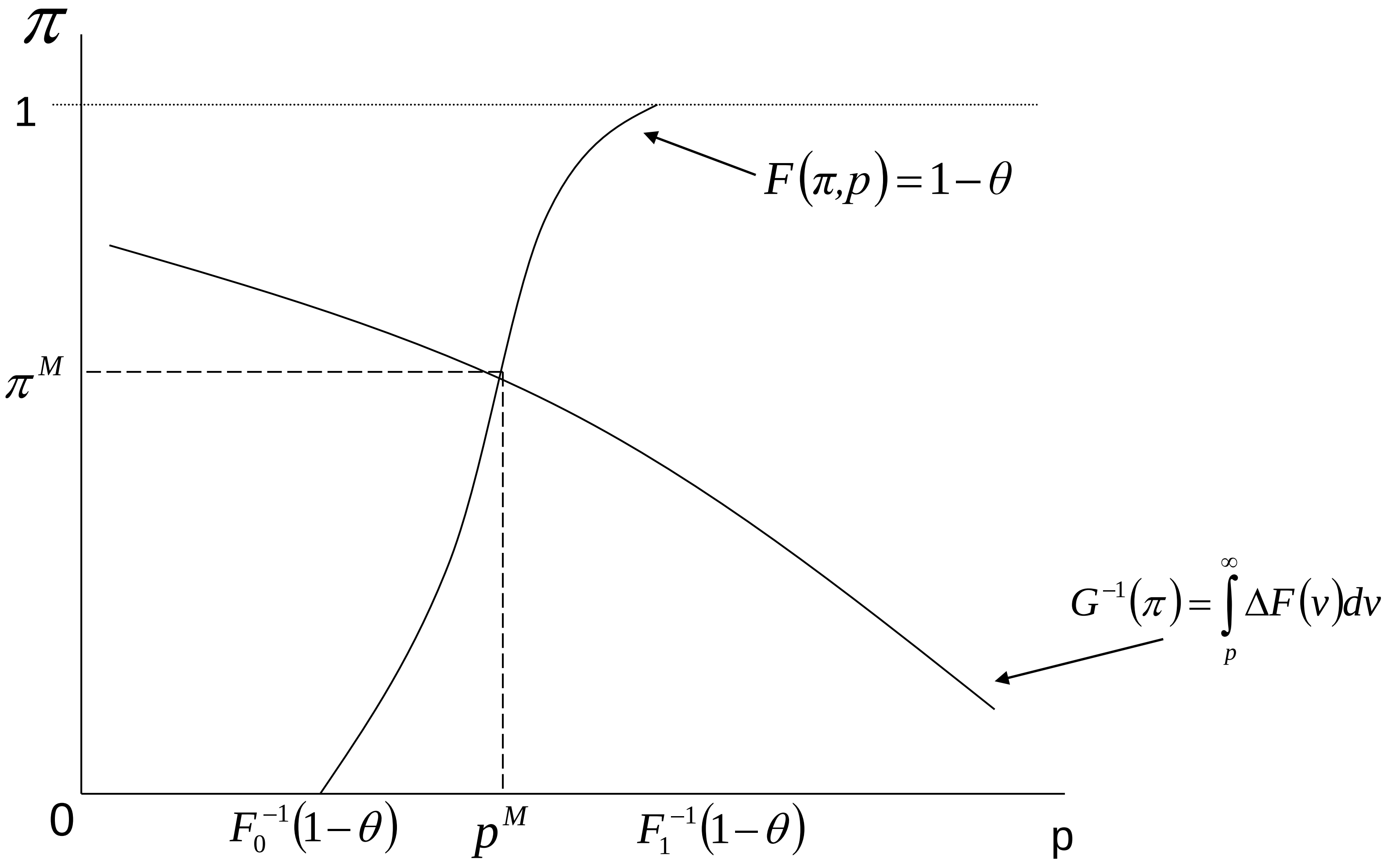


Figure 2: Competitive Equilibrium under Laissez-faire

(B) Efficiency: A more general formulation of the resource allocation problem: choose a feasible allocation of HC and of slots so as to maximize net social surplus, as follows:

An *allocation* is a pair of functions $e(i,c) \in \{0,1\}$ and $a(i,\mu) \in [0,1]$ specifying, respectively, HC investment of a group i agent with costs c, and the probability of obtaining a slot for a group i agent with productivity μ.

An allocation is *feasible* if:

$$\sum_{i\in\{a,b\}} \lambda_i \int_0^\infty a(i,\mu) f(\pi_i,\mu) d\mu \leq \theta$$

where

$$\pi_i \equiv \int_0^\infty e(i,c) g_i(c) dc$$

That is, π_i is the fraction of group i agents who invest in human capital. An allocation is feasible if it does not assign more agents to slots than there are slots available.)

Finding the efficient allocation under LF:

Now, the gross output from some feasible allocation is given by:

$$Q \equiv \sum_{i \in \{a,b\}} \lambda_i \int_0^\infty \mu a(i,\mu) f_i(\pi_i,\mu) d\mu$$

And the aggregate HC investment cost incurred with that allocation is:

$$C \equiv \sum_{i \in \{a,b\}} \lambda_i \int_0^\infty ce(i,c) g_i c) dc$$

So, an efficient allocation solves the problem:

$$Max\{Q - C\} \text{ subject to } \{e(i,c), a(i,\mu)\} \text{ feasible.}$$

Clearly, in an efficient allocation only the most productive agents get slots and only the lowest cost agents acquire HC. So, such an allocation is completely determined by the associated aggregate HC investment rate, π. So:

Given $\pi \in [0,1]$.and $0 < \theta < 1$, aggregate of widget values is at most:

$$Q(\pi, \theta) \equiv \int_{1-\theta}^{1} F^{-1}(\pi, z)dz;$$

and the aggregate of effort costs is at least:

$$C(\pi) \equiv \int_{0}^{\pi} G^{-1}(z)dz.$$

So, efficiency requires:

$$\pi^* \equiv \arg\max_{0 \leq \pi \leq 1} \{Q(\pi, \theta) - C(\pi)\}$$

First-order condition:

$$G^{-1}(\pi^*) = \int_{1-\theta}^{1} \frac{\partial F^{-1}}{\partial \pi}(\pi^*, z)dz \quad = \int\limits_{F^{-1}(\pi^*,1-\theta)}^{\infty} \Delta F(v)dv.$$

compared to equilibrium condition:

$$p^m = F^{-1}(\pi^m, 1-\theta) \text{ and } \pi^m = G(\int_{p^m}^{\infty} \Delta F(v)dv)$$

Conclude:

$$\pi^m = \pi^*$$

and

$$\int_{p^m}^{\infty} (v - p^m)(f_1(v) - f_0(v))dv = \frac{\partial Q}{\partial \pi}(\pi^*, \theta).$$

Group inequality under Laissez-faire:

$$\pi_i^* = G_i\left(\int_{F^{-1}(\pi^*,1-\theta)}^{\infty} \Delta F(v)dv\right) \text{ and}$$
$$\rho_i^* = 1 - F(\pi_i^*, F^{-1}(\pi^*, 1-\theta)),\ i = a, b$$

So, Assumptions 1 and 2 imply:

$$\pi_a^* > \pi^* > \pi_b^* \text{ and } \rho_a^* > \theta > \rho_b^*.$$

Proposition 1: *The equilibrium allocation of resources under laissez faire is socially efficient. Given our assumptions, the disadvantaged group (B) exerts productivity-enhancing effort, and gains access to productive opportunities, at a lower rate than does the advantaged group (A) in this equilibrium.*

Sighted Affirmative Action:

- Transfers contingent on actions {effort $\times$ slot $\times$ identity}
- Normalize such that transfer is zero if $e = 0$ or $\alpha = 0$ (WLOG)
- Also normalize such that transfer is zero if $e = 1$ and $i = A$. (WLOG)

Therefore, in a sighted environment an affirmative action policy is fully determined by three numbers – $(\sigma_a, \sigma_b, \tau)$ – denoting, respectively, the regulator's transfers to the A's and B's who exert effort, and to the B's who hold slots.

The key point is that, given the representation constraint, net social surplus depends entirely on the group specific rates of HC investment, π_a and π_b. So, the constrained optimal AA policy is the one which induces groups to acquire HC at the surplus-maximizing, group-specific rates.

Given (π_a, π_b) there is unique sighted policy generating this in equilibrium:

$$\sigma_i = G_i^{-1}(\pi_i) - \int_{F^{-1}(\pi_i, 1-\rho_i)}^{\infty} \Delta F(v) dv, \quad i = a, b; \text{ and}$$

$$\tau = F^{-1}(\pi_a, 1 - \rho_a) - F^{-1}(\pi_b, 1 - \rho_b)$$

Hence, without any further loss of generality we may characterize optimal sighted AA policy, both ex ante and ex post, by finding the pair of group-specific investment rates which maximize net social surplus subject to the capacity and representation constraints, and then by "backing out" the action-contingent transfers needed to induce the lowest cost agents in each group to invest at these optimal rates. But, this implies that the optimal policy entails no ex ante subsidy for investment by members of the disadvantaged group!

So, under sighted affirmative action, regulator maximizes:

$$\text{sighted surplus} = \sum_{i=a,b} \lambda_i \{Q(\pi_i, \rho_i) - C_i(\pi_i)\},$$

Implying::

$$G_i^{-1}(\pi_i^s) = \int_{F^{-1}(\pi_i^s, 1-\rho_i)}^{\infty} \Delta F(v) dv, \; i = a, b.$$

which means:

Proposition 3 *Given a representation target $\rho_b \in (\rho_b^*, \theta]$, with $\rho_a \equiv [\theta - \lambda_b \rho_b]/\lambda_a$, let (π_a^s, π_b^s) be the unique solutions above. Then, the efficient sighted affirmative action policy is:*

$$\begin{aligned} \sigma_a^s &= \sigma_b^s = 0 \text{ and} \\ \tau^s &= F^{-1}(\pi_a^s, 1 - \rho_a) - F^{-1}(\pi_b^s, 1 - \rho_b) \end{aligned}$$

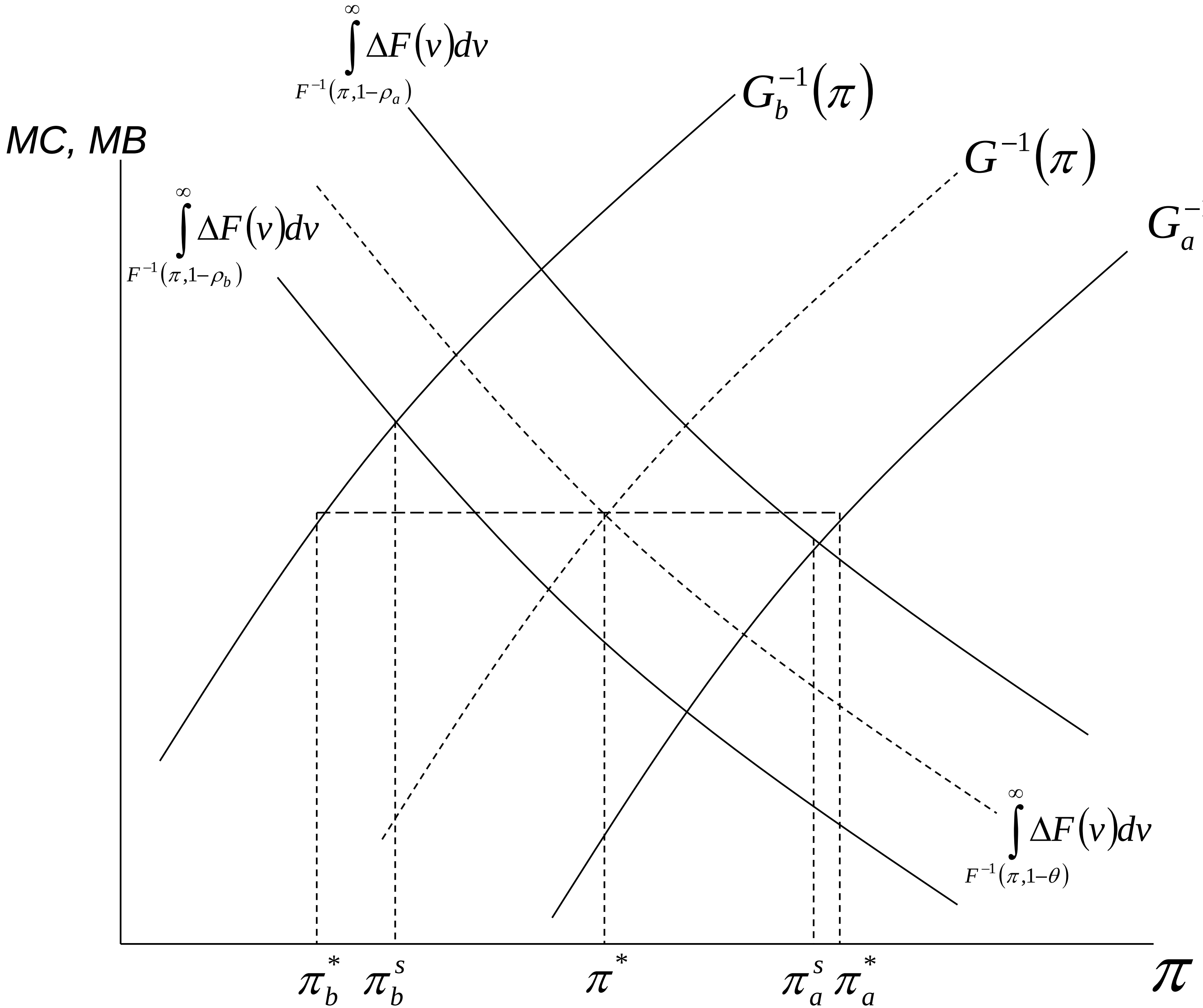


Figure 3: Optimal Sighted Affirmative Action

“Blind” Affirmative Action: The Assignment Margin

Given a worker’s productivity, μ, and given the group-specific HC investment rates, π_a and π_b, let $\xi(\mu)$ denote the probability that this worker belongs to group B. That is:

$$\xi(\mu) \equiv \frac{\lambda_b f(\pi_b, \mu)}{f(\pi, \mu)}.$$

Assumption 2 implies that $\xi(\mu)$ is strictly decreasing when $\pi_a > \pi_b$
We further assume that $\xi(\mu)$ is strictly convex.

Given these definitions, the socially efficient blind ex post slot assignment policy, denoted $\bar{a}(\mu)$, is characterized by the following infinite-dimensional linear program:

$$\{\bar{a}(\mu)\}_{\mu\geq 0} \equiv \arg\max_{\{a(\mu)\}} \left\{ \int_0^{\infty} \mu\, a(\mu) f(\pi,\mu)\, d\mu \text{ subject to} \right. \tag{6}$$
$$\left. \int_0^{\infty} \xi(\mu)\, a(\mu) f(\pi,\mu)\, d\mu = \lambda_b \sigma_b \in (\lambda_b \sigma_b^e, \lambda_b \theta] \right\}$$

(representation constraint),

$$\int_0^{\infty} a(\mu) f(\pi,\mu)\, d\mu = \theta$$

(capacity constraint), and

$$a(\mu) \in [0, 1] \text{ is nondecreasing in } \mu.$$

Let σ_a and σ_b be the target slot-holding rates of groups A and B, respectively, and let $\pi_a > \pi_b$ be the groups' rates of HC acquisition.

Define $\bar{p}(\pi_a, \pi_b)$ as follows:

$$\frac{F(\pi_a, \bar{p})}{F(\pi_b, \bar{p})} \equiv \frac{1 - \sigma_a}{1 - \sigma_b}.$$

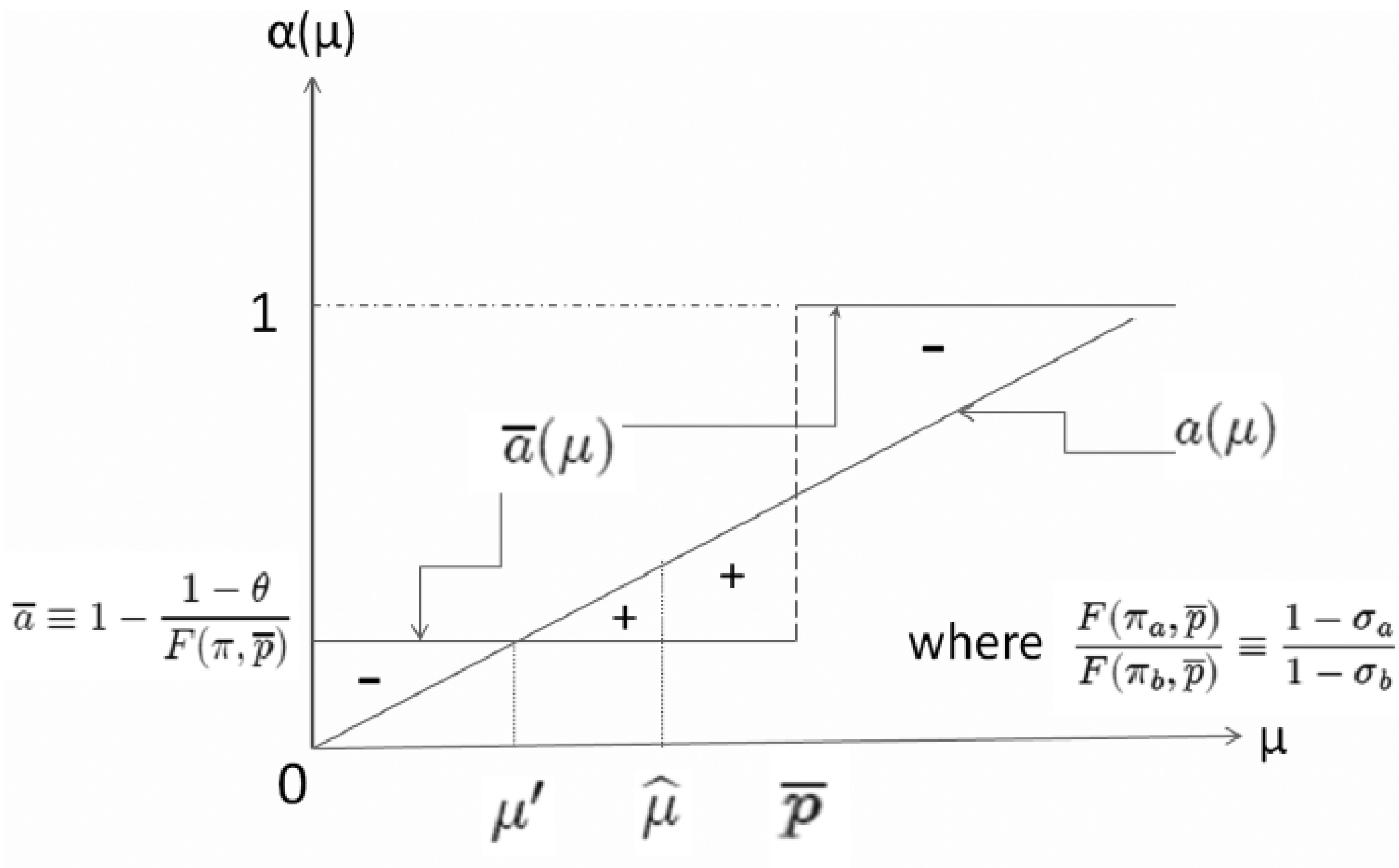
Finally, given π and $\bar{p}$, define the number $\bar{a}$ such that, if the measure of slots $\bar{a}$ were to be given away to the population at random while the remaining measure of slots $(\theta - \bar{a})$ were sold to the highest remaining bidders, then the market-clearing price for these remaining slots would be $\bar{p}$. That is, let $\bar{a}$ be defined by the following equation:

$$\text{demand} = (1 - \bar{a})[1 - F(\pi, \bar{p})] \equiv \theta - \bar{a} = \text{supply}.$$

Given these definitions, we have the following characterization of optimal “blind” AA policy at the assignment margin:

THEOREM 1. Given the group-specific ex ante investment rates, $\pi_a > \pi_b$, under the MLRP assumptions on $F_e(\mu)$ and $G_i(c)$, and with the additional assumption that $f_1(\mu)/f_0(\mu)$ is concave in μ, then, for $\bar{a}$ and $\bar{p}$ as defined above, the solution to the ex post linear optimization problem under blind affirmative action, $\{\bar{a}(\mu)\}_{\mu\geq 0}$, is given as follows:

$$\bar{a}(\mu) = \begin{cases} \bar{a} \equiv 1 - \dfrac{1-\theta}{F(\pi, \bar{p})} & \text{if } \mu \leq \bar{p} \\ 1 & \text{if } \mu > \bar{p}. \end{cases}$$



(This diagram may be used to establish an informal “proof” of Theorem 1)

(3) Using College Admissions Data to Simulate the Impact of Imposing "Color-Blindness"

An Economic Analysis of Color-Blind Affirmative Action

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The Journal of Law, Economics, & Organization, Vol. 24, No. 2,
doi:10.1093/jleo/ewm053
Advance Access publication December 24, 2007

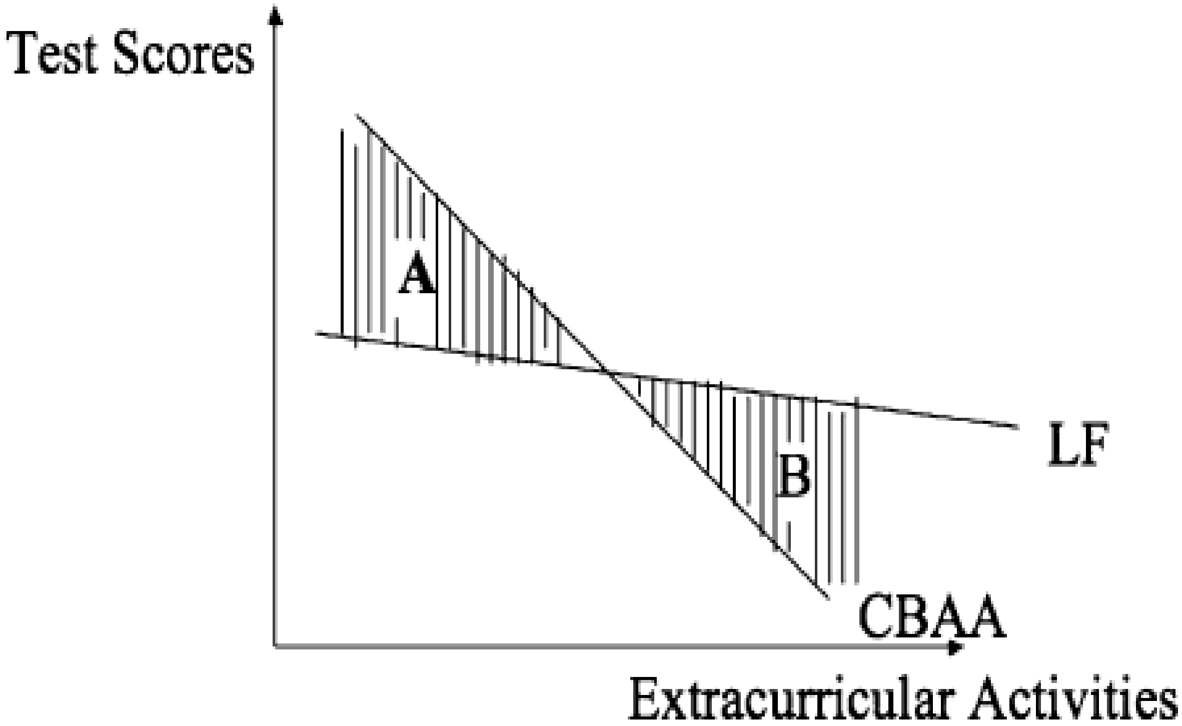
Affirmative Action without Explicit Racial Discrimination

- Color-blind (non-racially discriminatory) affirmative action exploits statistical associations in the population between an applicant's racial identity and his/her non-racial traits

[Texas 10% Plan famously illustrates the non-transparency]

- A policymaker alters the weight given to non-racial traits for all applicants in such a way as to increase the yield in selection process from a targeted group.
- One consequence of this kind of policy is that selection efficiency must in general be reduced for all applicants. Policy can't be 'conditionally' (within group) meritocratic.

Illustrating How Color-Blind Affirmative Action Works:



LF is the “admissions frontier” under laissez-faire; CBAA is frontier under a color-blind affirmative action policy. This policy excludes applicants in area A and includes those in area B. Suppose the total number of applicants in the two areas are the same, but extracurricular activities are distributed more similarly between the groups than test scores. Then more disadvantaged group students will fall into area B than into area A. Hence, the policy will increase the number of disadvantaged students admitted at the cost of reducing expected academic performance of the admitted class.

Simulating consequences of CBAA for college admissions

In what follows, $i \in I$ denotes an applicant; x_i is applicant i's vector of traits; p_i is the expected post-admissions performance of applicant i; r_i is the probability applicant I belongs to some targeted group; and (β, γ) are (to be estimated) coefficient vectors that relate applicant traits to performance and racial identity, respectively.

An admissions planner is imagined to choose an admission probability A_i for each applicant in so as to maximize the expected performance of the admitted class subject to a capacity ($0<c<1$) and a representation ($0<r<1$) constraint.

We can use C&B data to estimate two (presumed linear) relationships:

(1) Academic Performance Equation (p=college grades):

$$p_i \equiv [\text{Expected performance} \mid x_i] = \beta \cdot x_i = \sum_{j \in J} \beta_j x_i^j$$

(2) Racial Identity Equation (r=applicant's race):

$$r_i \equiv \Pr[R_i = 2 \mid x_i] = \gamma \cdot x_i = \sum_{j \in J} \gamma_j x_i^j$$

With those estimates of the coefficients β and γ in hand, we calculate the optimal policy by solving this simple LP problem:

$$\max_{[A_i]_{i\in I}} \left\{ \left(\frac{1}{c}\right) \sum_{i\in I} A_i p_i \right\}, \quad \text{subject to the following three constraints:}$$

$$\text{(i)} \ A_i \in [0,1], \ i \in I, \quad \text{(ii)} \ \frac{1}{|I|}\left\{\sum_{i\in I} A_i\right\} \le c, \quad \text{(iii)} \ \frac{1}{|I|}\left\{\sum_{i\in I} A_i r_i\right\} \ge r.$$

(1) Laissez-Faire Solution: **Ignoring representation constraint implies a threshold rule on Predicted Performance, for some μ > 0**

$$A_i^* = \begin{cases} 1 & \text{if } \beta \cdot x_i > \mu \\ 0 & \text{if } \beta \cdot x_i < \mu. \end{cases}$$

Here μ must be chosen in such a way that constraint (ii) holds with equality.

***(2) Color-Sighted Affirmative Action Solution:* Allowing explicitly discriminatory admissions implies two race-specific thresholds**

Under the CS regime, there will be separate thresholds for the racial groups So, for a pair of numbers μ_1 and μ_2, with $\mu_1 > \mu_2$, we have:

$$A_i^* = \begin{cases} 1 & \text{if } \beta \cdot x_i > \mu_{R_i} \\ 0 & \text{if } \beta \cdot x_i < \mu_{R_i}. \end{cases}$$

Here the μ_1 and μ_2 are to be chosen such that selection rates for the two groups are consistent with the capacity and representation constraints holding as equalities.

***(3) Color-Blind Affirmative Action Solution:* A common threshold for both groups is employed, but with suitably modified weights in scoring equation. So, for some numbers θ>0 and μ´>0, we have the following:**

Under the CB regime, a Lagrangian multiplier on constraint (iii) alters the admissions policy relative to LF because nonracial traits are now to be valued both for their association with prospective academic performance and for their ability to predict an applicant's race. Thus, the optimal CB policy is characterized by two numbers θ and μ' such that:

$$A_i^* = \begin{cases} 1 & \text{if } [\beta + \theta\gamma] \cdot x_i > \mu' \\ 0 & \text{if } [\beta + \theta\gamma] \cdot x_i < \mu', \end{cases}$$

where μ' and θ are such that constraints (ii) and (iii) above hold as equalities.

Notice that CBAA distorts an applicant's incentives to acquire traits that enhance post-admissions performance (relative to CSAA):

activities and test scores). Under LF and CS regimes, the college's marginal rate of substitution between traits j and k as reflected in the admissions policy function, denoted by $MRS_{j,k}$, is equal to the relative importance of these traits in forecasting student performance:

$$MRS_{j,k} = \frac{\beta_j}{\beta_k},$$

whereas, under the CB regime, the rate of substitution between traits j and k that holds constant the probability of being admitted is given by:

$$MRS_{j,k} = \frac{\beta_j + \theta\gamma_j}{\beta_k + \theta\gamma_k}.$$

Admissions “Experiment”: CSAA versus CBAA

To get a feel for the quantitative effects of color-blind policy, as compared to color-sighted AA, we take the College and Beyond data set (on admitted students at a number of selective US colleges) in order to estimate relationships between non-racial applicant traits, the racial identity of applicant (γ), and applicants’ post-admissions performance (β).

We can simulate consequences of a hypothetical “admissions experiment” by supposing each college must reduce its admitted class by half while keeping the proportion of black students in this reduced class at the same level it had been among those who were actually admitted. (So, $c=0.5$) We then compare the expected performance of the hypothetically reduced classes under both the CBAA and CSAA scenarios.

Estimates of β

Table 3. Performance Equation: Predicted College Rank

	College A	College B	College C	College D
SAT math	4.04 (1.39)	−0.60 (1.78)	5.08 (1.69)	7.57 (1.51)
SAT verbal	5.47 (1.31)	8.98 (1.62)	7.15 (1.68)	12.85 (1.30)
HS percentile	3.12 (1.11)	8.79 (1.42)	8.42 (1.84)	7.49 (1.79)
Mother college educated	2.58 (2.63)	8.40 (3.00)	−3.45 (3.76)	3.61 (2.15)
Father college educated	4.35 (2.99)	−3.76 (3.60)	6.07 (4.04)	5.48 (2.84)
Zip income	−0.04 (0.64)	−1.44 (0.80)	−0.47 (0.72)	−0.74 (0.44)
Legacy	4.66 (4.55)	0.59 (4.05)	0.65 (3.64)	−0.47 (1.96)
Percent Asian in zip	14.07 (16.83)	16.78 (16.82)	6.28 (19.45)	33.05 (13.58)
Percent Black in zip	−11.72 (5.78)	−29.10 (10.99)	−14.26 (7.59)	−15.91 (5.31)
Percent Hispanic in zip	−15.76 (11.21)	−22.15 (11.51)	−0.42 (11.40)	−3.24 (9.10)
Male	−4.77 (2.06)	n/a	n/a	−7.92 (1.66)
R^2	0.16	0.21	0.19	0.37
Number of observation	761	429	512	494

College rank is percentiles in distribution of cumulative GPA among students who matriculated at that college in 1989. HS percentile is studen
dummies for students' mother and father being college educated. Zip income is the median income of the student's zip code from the 1990 Ce
percentile 10 percentiles, ZIP income $10,000. We used dummies for the missing data. (Coefficients for these variables are not reported ir

Estimates of γ

Table 4. Race Equation: Probability of Being Black

	College A	College B	College C	College D
SAT math	−0.06 (0.01)	−0.06 (0.01)	−0.07 (0.01)	−0.10 (0.02)
SAT verbal	−0.02 (0.01)	−0.03 (0.01)	−0.06 (0.01)	−0.04 (0.01)
HS percentile	−0.04 (0.01)	0.01 (0.01)	−0.07 (0.01)	−0.05 (0.01)
Mother college educated	0.02 (0.02)	0.01 (0.02)	0.03 (0.03)	0.01 (0.02)
Father college educated	−0.11 (0.02)	−0.04 (0.02)	−0.08 (0.03)	−0.02 (0.03)
Zip income	0.01 (0)	0.01 (0.01)	0 (0.01)	0.01 (0.01)
Legacy	0.01 (0.03)	−0.02 (0.03)	−0.01 (0.03)	−0.09 (0.03)
Percent Asian in zip	−0.25 (0.12)	−0.06 (0.11)	−0.10 (0.15)	0.14 (0.20)
Percent Black in zip	0.57 (0.04)	0.57 (0.07)	0.85 (0.06)	0.57 (0.06)
Percent Hispanic in zip	−0.02 (0.08)	−0.02 (0.08)	−0.08 (0.09)	−0.13 (0.12)
Male	0.01 (0.02)	n/a	n/a	0.01 (0.02)
R^2	0.41	0.22	0.55	0.37
Number of observations	761	429	512	494

Dependent variable is student's probability of being black. HS percentile is students' percentile in his high school. Mother's and father's edu educated. Zip income is the average income of the student's zip code from the 1990 Census; n/a, not available. Increments: SAT variables 100 dummies for the missing data. (Coefficients for these variables are not reported in this table.)

Predicted relative performance of selected students after reducing actual class by half

Table 5. Relative Performances of Color-Blind and Color-Sighted Policies, by Race Constraint

	College A	College B	College C	College D	College E	College F	College G	Average
Random admissions	86.29	87.22	80.43	85.04	80.67	85.62	81.26	83.62
Laissez-faire without SAT	96.96	98.50	97.00	93.90	95.01	98.45	96.65	96.88
Laissez-faire without HS percentile	99.77	98.02	97.89	99.58	99.56	98.67	97.24	98.57
Color sighted	97.97	99.55	97.64	97.43	98.66	98.59	99.77	98.68
Color blind	94.28	98.67	95.33	90.82	96.40	95.95	98.74	96.16

Predicted college rank of a student is estimated by the OLS regression. For each policy, we compute the average predicted college rank of the admitted class. We call this value the performance of the policy. To compute the relative performance, we index laissez-faire's performance as 100. For example, color-sighted relative performance = (color-sighted performance × 100)/(laissez-faire performance). Average is the population-weighted average.

"Blindness" Alters Selection Formula, reducing weight given to academic factors and increasing emphasis on social background: [β to ($\beta + \theta\gamma$)]

Table 7. Weight on Students' Characteristics in the Admission Formula for Laissez-Faire and Color-Blind Policies,

	SAT math		SAT verbal		HS percent		Mother educated		Father educated		Income		Percer black	
	LF	CB	LF	CB	LF	CB	LF	CB	LF	CB	LF	CB	LF	
College A	4.04	0.16	5.47	4.18	3.12	0.53	2.58	3.87	4.35	−2.76	−0.04	0.61	−11.72	2
College B	−0.60	−4.88	8.98	6.84	8.79	9.50	8.40	9.11	−3.76	−6.61	−1.44	−0.73	−29.10	1
College C	5.08	1.68	7.15	4.24	8.42	5.02	−3.45	−1.99	6.07	2.18	−0.47	−0.47	−14.26	2
College D	7.57	−3.74	12.85	8.33	7.49	1.83	3.61	4.74	5.48	3.22	−0.74	0.39	−15.91	1